Replica field theory for a polymer in random media

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In this paper we revisit the problem of a (non-self-avoiding) polymer chain in a random medium which was previously investigated by Edwards and Muthukumar (EM) [J. Chem. Phys. **89**, 2435 (1988)]. As noticed by Cates and Ball (CB) [J. Phys. (France) **49**, 2009 (1988)] there is a discrepancy between the predictions of the replica calculation of EM and the expectation that in an infinite medium the quenched and annealed results should coincide (for a chain that is free to move) and a long polymer should always collapse. CB argued that only in a finite volume one might see a ''localization transition'' (or crossover) from a stretched to a collapsed chain in three spatial dimensions. Here we carry out the replica calculation in the presence of an additional confining harmonic potential that mimics the effect of a finite volume. Using a variational scheme with five variational parameters we derive analytically for $d<4$ the result $R \sim (g|\ln \mu|)^{-1/(4-d)} \sim (g \ln V)^{-1/(4-d)}$, where R is the radius of gyration, g is the strength of the disorder, μ is the spring constant associated with the confining potential, and V is the associated effective volume of the system. Thus the EM result is recovered with their constant replaced by $\ln V$ as argued by CB. We see that in the strict infinite volume limit the polymer always collapses, but for finite volume a transition from a stretched to a collapsed form might be observed as a function of the strength of the disorder. For $d<2$ and for large $V>V' \sim \exp(g^{2/(2-d)}L^{(4-d)/(2-d)})$ the annealed results are recovered and $R \sim (Lg)^{1/(d-2)}$, where L is the length of the polymer. Hence the polymer also collapses in the large L limit. The one-step replica symmetry breaking solution is crucial for obtaining the above results.

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I. INTRODUCTION

There has been much interest in recent years in the properties of polymer chains in a quenched random environment [1–5]. This problem is directly related to that of a quantum particle in a random medium [6,7] and to that of a flux line in a type II superconductor in the presence of random columnar defects [8,9] as will be made clear below. Thus its general application makes it important for a variety of physical situations.

The quantities of interest for the polymer problem are the free energy and the radius of gyration of a chain in a quenched white-noise potential. Here we consider only the case of a non-self-avoiding chain. Cates and Ball [2] gave a beautiful intuitive argument to the effect that a Gaussian chain situated in an infinite random medium is always collapsed in the long-chain limit. Their argument goes as follows: Consider a white-noise random potential $v(x)$ of zero mean whose probability distribution at each site is

$$P[v(x)] \propto g^{-1/2} \exp(-v^2/2g). \tag{1.1}$$

If we now coarse-grain the medium and denote by $\bar{v}$ the average value of the potential over some region of volume a, Then the coarse-grained potential will have the distribution

$$P_a(\bar{v}) \propto (g/a)^{-1/2} \exp(-a\bar{v}^2/2g). \tag{1.2}$$

Consider a polymer chain situated in the random potential, and assume that it shrinks into a volume a corresponding to a place where the mean potential $\bar{v}$ takes on a lower value

than usual. In this situation the free energy of the chain is crudely estimated to be (neglecting all numerical factors):

$$F(a,\bar{v}) = L/R^2 + L\bar{v} + a\bar{v}^2/2g. \tag{1.3}$$

Here L is the length of the chain (number of monomers), R is the radius of gyration (or end to end distance) and the volume a is related to R via $a = R^d$ in d-spatial dimensions. The first term on the right-hand side (RHS) is an estimate of the free energy of a long chain confined to a region of size R in the absence of an external potential [see, e.g., Ref. [10], Eq. (I.12)]. The second term is just the potential energy of the chain in the random potential of strength $\bar{v}$. The third term arises from the chance of incurring a random potential of strength $\bar{v}$. The quantity $\ln P(\bar{v})$ gives an associated effective entropy for the system. Minimizing this free energy over both $\bar{v}$ and a determines the lowest free energy configuration. Minimizing with respect to $\bar{v}$ yields $\bar{v} = -Lg/a$. Substituting in F gives

$$F(R) = \frac{L}{R^2} - \frac{L^2 g}{2R^d}. \tag{1.4}$$

This shows that for any $d \geq 2$, $F \to -\infty$ as $R \to 0$. Thus the mean size of the chain is zero, or in the presence of a cutoff of a size of one monomer,

$$R \sim 1, \quad d \geq 2. \tag{1.5}$$

For $d<2$, the free energy has a minimum for

$$R \sim (Lg)^{1/(d-2)} \quad d<2, \tag{1.6}$$

which in the long chain limit ($L \gtrsim 1/g$) cuts off again at $R \sim 1$. These results are the same as those for the case of an annealed potential that is able to adjust locally to lower the free energy of the system. The reason is that for an infinite system containing a finite (even though long) chain, space can be divided into regions containing different realizations of the potential, and the chain can sample all of these to find an environment arbitrarily similar to that which would occur in the annealed situation.

These results stand in contrast to the replica calculation of Edwards and Muthukumar (EM) [1], who found that for a long chain

$$R \sim g^{-1/(4-d)}, \quad d < 4 \qquad (1.7)$$

when $g^{2/(4-d)}L \to \infty$, whereas $R \sim L^{1/2}$ when $g^{2/(4-d)}L \to 0$. Note that the result (1.7) is independent of L as opposed to Eq. (1.6). To reconcile the two apparently different results, Cates and Ball argue that the quenched case is different from the annealed case only for the case when the medium has a *finite* volume V. In a finite box, arbitrarily deep potential minima are not present. Instead the most negative $\bar{v}$ averaged over a region of volume $a \ll V$ occupied by the chain, is approximately (keeping only leading terms in the volume V) given by solving the equation [the left-hand side (LHS) of which represents the area under the tail of the distribution]

$$\int_{-\infty}^{\bar{v}} dy \, P_a(y) \simeq \frac{a}{V}, \qquad (1.8)$$

which yields

$$\bar{v} = -\sqrt{\frac{g \ln V}{a}}. \qquad (1.9)$$

This expression when plugged into Eq. (1.3) leads to [note that the last term in Eq. (1.3) just becomes a constant independent of R]

$$F(R) = \frac{L}{R^2} - L \sqrt{\frac{g \ln V}{R^d}}. \qquad (1.10)$$

When this free energy is minimized with respect to R it gives rise to

$$R \sim (g \ln V)^{-1/(4-d)}, \quad d < 4 \qquad (1.11)$$

which agrees with Eq. (1.7) and also with simulations performed on a chain in a random medium of a fixed finite volume [4]. However, it is not clear from this explanation why the replica calculation which has been done for an infinite system [1] gives rise to the finite volume result. To shed light on this question we will show in this paper that the reason for the discrepancy is the fact that EM used a variational calculation which relies on a single variational parameter. We show specifically that the single parameter variational solution is inconsistent.

What we will do first is, instead of considering a system in a finite volume which is hard to solve, introduce an external harmonic potential (with a spring constant μ). Such an attractive potential has the effect of confining the chain to a finite distance from the origin since the energy cost to wan-

der far away from the origin of the potential is high. A system in a harmonic potential is easier to solve than a system in a finite box. It also corresponds directly to the problem of a flux line in a type II superconductor where the cage potential felt by a flux line due to its neighboring flux lines can be modeled by a harmonic potential (see below). In addition, we introduce more variational parameters, three for the case of replica symmetric parametrization and five for the case of replica symmetry breaking (RSB). These extra parameters have physical significance as will be discussed below. We will then use the replica method and the variational approximation to tackle the problem and obtain the free energy and the radius of gyration. For finite μ we find that R is independent of L (the chain length) and as the disordered strength is increased from zero, R is decreased from its initial μ-dominated value according to the relation $R \sim (g|\ln \mu|)^{-1/(4-d)}$ [which agrees with Eq. (1.11) since the effective volume available to a system in a harmonic potential is $\ln V \sim |\ln \mu|$].

On the other hand, if we try to take the ultimate $\mu \to 0$ limit (which is the case originally studied by EM), the previous solution becomes invalid and the chain collapses for $d \geq 2$. For $d < 2$ the annealed results are obtained in the $\mu \to 0$ limit as given above in Eqs. (1.4),(1.5),(1.6). This occurs specifically because of the extra variational parameters used beyond the single variational parameter used by EM. We also demonstrate the importance of RSB for obtaining the correct physical results. [The relevance of RSB to this model was recognized by Haronska and Vilgis [5], but unfortunately their calculation still predicted a constant coefficient of proportionality in the relation $R \sim (cg)^{-1/(4-d)}$, that although differs from the EM result does not contain the correct $\ln V$ dependence.]

To define the model of a polymer chain in a random potential plus a fixed harmonic potential we use the Gaussian chain approximation to write

$$H = \int_0^L du \left[\frac{M}{2} \left(\frac{\partial \mathbf{R}(u)}{\partial u} \right)^2 + \frac{\mu}{2} \mathbf{R}^2(u) + V(\mathbf{R}(u)) \right], \qquad (1.12)$$

where $\mathbf{R}(u)$ is the d-dimensional position vector of the chain at arc length u ($0 \leq u \leq L$), μ governs the strength of the harmonic potential and $V(\mathbf{R})$ is the random potential satisfying:

$$\langle V(\mathbf{R}) \rangle = 0, \quad \langle V(\mathbf{R}) V(\mathbf{R}') \rangle = g \, \delta^{(d)}(\mathbf{R} - \mathbf{R}'). \qquad (1.13)$$

We can actually consider a wider class of random potential correlations characterized by a function f:

$$\langle V(\mathbf{R}) V(\mathbf{R}') \rangle = g \, df \left(\frac{(\mathbf{R} - \mathbf{R}')^2}{d} \right), \qquad (1.14)$$

where $f()$ is some given function. In Eq. (1.12) we choose the units such that u is dimensionless and so L is the length of the polymer in units of the Khun bond step b. The ''mass'' M is inversely proportional to βb^2, where $\beta = 1/k_B T$ (in d dimensions $\beta M = d/b^2$). The case $\mathbf{R}(0) = \mathbf{R}(L)$ corresponds to a closed chain.

The partition sum is given by the functional integral

$$Z(\mathbf{R}, \mathbf{R}', L, \beta) = \int_{\mathbf{R}(0)=\mathbf{R}}^{\mathbf{R}(L)=\mathbf{R}'} \mathcal{D}\mathbf{R}(u) \exp(-\beta H). \quad (1.15)$$

We further define a boundary-free partition sum (for a closed chain) by

$$Z(L, \beta) = \int d\mathbf{R} \; Z(\mathbf{R}, \mathbf{R}, L, \beta) \quad (1.16)$$

and the free energy is given by

$$\beta F = -\ln Z(L, \beta). \quad (1.17)$$

The correlation function of interest is

$$C(\ell) = \frac{1}{d} \langle\langle [\mathbf{R}(\ell) - \mathbf{R}(0)]^2 \rangle\rangle_R, \quad (1.18)$$

where

$$1 \ll \ell \ll L. \quad (1.19)$$

The first average in Eq. (1.18) is the thermal one with a Boltzmann weight $\exp(-\beta H)$ and the second average is over the realizations of the random potential. For the range of ℓ given by Eq. (1.19), the boundary conditions on the chain, e.g., open or closed are not important for the behavior of $C(\ell)$.

For the case of no disorder (i.e., $g=0$) the correlation function is given by

$$C_0(\ell) = \frac{1}{\beta\sqrt{M\mu}} [1 - \exp(-\ell\sqrt{\mu/M})]. \quad (1.20)$$

We see that in the limit $\mu \to 0$, $C_0(\ell) \sim \ell/\beta M \sim (b^2/d)\ell$, which corresponds to pure diffusion of the chain (random walk). From the relation

$$\langle\langle \mathbf{R}(0)^2 \rangle\rangle_R = \frac{d}{\beta\sqrt{M\mu}}, \quad (1.21)$$

we see that the polymer chain is confined to a volume of size V satisfying

$$\ln V \sim \frac{d}{4} |\ln \mu|, \quad (1.22)$$

for small μ.

The mapping of this problem to a vortex line in an harmonic cage potential and random columnar defects is such that the arc-length u corresponds to the distance z along the c axis (assuming this is also the direction of the magnetic field), $M \to \epsilon_l = \epsilon_0/\gamma^2$, which is the line tension of the flux line and $\gamma^2 = m_z/m_\perp$ is the mass anisotropy. $\mathbf{R}$ is a two-dimensional vector in the a-b plane of the superconductor [8,9]. The harmonic potential plays an essential role as a reasonable approximation to the cage potential that a vortex line feels due to the repulsion by its neighbors. Thus $\mu \approx \epsilon_0 B/\Phi_0$ where B is the magnetic field and Φ_0 is the fluxoid.

There is also a mapping into the problem of a quantum particle in a random potential + a harmonic potential. This mapping reads [11,6]

$$\beta \to 1/\hbar, \quad L \to \beta\hbar, \quad (1.23)$$

and $\rho(\mathbf{R}, \mathbf{R}', \beta) = Z(\mathbf{R}, \mathbf{R}', L = \beta\hbar, \beta = 1/\hbar)$ becomes the density matrix of a quantum particle at inverse temperature β. The variable u represents the Trotter (imaginary) time. In this case M corresponds to the mass of the particle.

II. VARIATIONAL CALCULATION

In order to average over the quenched random potential we use the replica method. After introducing n copies of the chain and averaging over the random potential one obtains

$$\langle Z^n \rangle = \int \mathcal{D}\mathbf{R}_1 \cdots \mathcal{D}\mathbf{R}_n \exp(-\beta H_n), \quad (2.1)$$

with

$$H_n = \int_0^L du \sum_{a=1}^n \left[\frac{M}{2}\left(\frac{\partial\mathbf{R}_a(u)}{\partial u}\right)^2 + \frac{\mu}{2}\mathbf{R}_a^2(u) \right]$$
$$- \frac{\beta g}{2} \int_0^L du \int_0^L du' \sum_{ab} \delta^{(d)}[\mathbf{R}_a(u) - \mathbf{R}_b(u')]. \quad (2.2)$$

Here we used the delta function potential (to make contact with EM), but later we will show how to generalize to a general correlation. It is useful to replace the delta function by the equivalent expression

$$\delta^{(d)}[\mathbf{R}_a(u) - \mathbf{R}_b(u')] = \int \frac{d\mathbf{k}}{(2\pi)^d} \exp\{i\mathbf{k}[\mathbf{R}_a(u) - \mathbf{R}_b(u')]\}. \quad (2.3)$$

For a general correlation [see Eq. (1.14)] we can write

$$f\{[\mathbf{R}_a(u) - \mathbf{R}_b(u')]^2/d\}$$
$$= \int d\mathbf{y} f(\mathbf{y}^2/d) \int \frac{d\mathbf{k}}{(2\pi)^d}$$
$$\times \exp(-i\mathbf{k}\cdot\mathbf{y}) \exp\{i\mathbf{k}[\mathbf{R}_a(u) - \mathbf{R}_b(u')]\}. \quad (2.4)$$

In order to proceed we use a quadratic variational Hamiltonian to be the best approximation to H_n. This is given by

$$h_n = \int_0^L du \sum_{a=1}^n \left[\frac{M}{2}\left(\frac{\partial\mathbf{R}_a(u)}{\partial u}\right)^2 + \frac{\mu}{2}\mathbf{R}_a^2(u) \right]$$
$$- \frac{1}{2} \int_0^L du \int_0^L du' \sum_{ab} q_{ab}(u-u')\mathbf{R}_a(u)\cdot\mathbf{R}_b(u'), \quad (2.5)$$

where $q_{ab}(u)$ are $n \times n$ variational functions to be determined, with $n \to 0$ at the end. The best variational Hamil-

tonian is determined by the stationarity of the variational free energy which is given by [11,1,12]

$$n\langle F\rangle_R = \langle H_n - h_n\rangle_{h_n} - \frac{1}{\beta}\ln\int \mathcal{D}\mathbf{R}_1 \cdots \mathcal{D}\mathbf{R}_n \exp(-\beta h_n).$$
(2.6)

The general equations satisfied by $q_{ab}(u)$ where discussed in Refs. [6,7,9]. We showed that although the diagonal elements $q_{aa}(u)$ must depend on the arc-length variable u, the off diagonal elements $q_{a\neq b}$ which are spin-glass-like order parameters can to be chosen to be u-independent; in other words there is a consistent solution of the variational equations with these properties. (The existence of a time-persistent part to the off diagonal elements of q_{ab} is well known in the investigation of quantum spin glass systems [13] and is crucial for the capture of the correct physics in such systems.) In Refs. [6,9] we proceeded to solve the equations approximately for the case of a nonzero confining harmonic potential characterized by a spring constant $\mu\neq 0$. For a quantum particle at not too low a temperature (equivalent for moderate values of L in the polymer problem) we obtained a numerical solution of the equations [6] for different types of correlations of the random potential. In Ref. [9] we considered the limit of large L and finite μ (in the context of the vortex line problem), and for $d=2$, under certain approximations obtained an analytical solution to first order in g (the strength of the disorder). Here we would like to consider the whole range of disorder for large L and also investigate the limit $\mu\to 0$. Our goal is also to make contact with the calculation of EM. Hence we will start with a somewhat simpler approach with a finite number of variational parameters in lieu of the infinite number of such parameters introduced in our previous work. As will turn out this is appropriate for the current problem and allows us to solve everything analytically without any further approximations.

EM considered only the case of $\mu=0$ and chose

$$q_{ab}(u-u') = -\frac{q^2 M}{9}\delta_{ab}\delta(u-u'),$$
(2.7)

where q is a single variational parameter. [In an appendix they considered a slightly more general form but it is still proportional to $\delta(u-u')$.] Here we claim that we need to introduce static (u-independent) off-diagonal elements for q_{ab} and also add a static diagonal part. This will help capture the correct physics of the problem as in the case of the quantum spin glass systems mentioned above. Thus we chose

$$q_{ab}(u-u') = -\delta_{ab}[(\lambda-\mu)\delta(u-u') + (\lambda_1-\lambda)/L] + (1-\delta_{ab})s/L,$$
(2.8)

and we have three variational parameters λ, λ_1, and s. The variables λ,λ_1 represent two values of $\lambda(\omega\neq 0)$ and $\lambda(\omega=0)$ instead of the general function $\lambda(\omega)$ introduced in Ref. [6] (which involves an infinity of variational parameters). The variable s represents a ''spin glass'' type variable which loosely speaking is a measure of ''freezing.''

At this point it is important to note that apart from the μ-dependent term, the Hamiltonian H_n given in Eq. (2.2) is invariant the transformation

$$\mathbf{R}_a(u)\to\mathbf{R}_a(u)+\mathbf{C}, \quad a=1,\ldots,n,$$
(2.9)

where $\mathbf{C}$ is a constant vector. This reflects the fact that in the infinite system, without a confining harmonic term, after averaging over the random potential the interaction is translational invariant. A good variational Hamiltonian must be one that preserves this translational symmetry. If we consider the variational Hamiltonian h_n as given by Eq. (2.5), where the μ-dependent part has been separated explicitly, the remaining interaction becomes translational invariant under the transformation (2.9) provided $q_{ab}(u)$ satisfies

$$\int_0^L du\sum_b q_{ab}(u) = 0.$$
(2.10)

Thus the variational ansatz must be rich enough to implement the relation (2.10). The ansatz chosen by EM as given by Eq. (2.7) violates the condition (2.10) for any $q\neq 0$. It imposes an unphysical origin on the system when none exists. We will see below [Eq. (2.48)] that with our variational parametrization as given in Eq. (2.8) the stationarity of the variational free energy gives rise to the condition (2.10). This is true even in the more general case of replica symmetry breaking, see Eq. (3.5) below. The condition (2.10) effectively reduces the number of variational parameters by one and will be crucial for obtaining the solution of the problem.

Substituting Eq. (2.8) in Eq. (2.5) for h_n, the variational Hamiltonian becomes

$$h_n = \int_0^L du\sum_{a=1}^n \left[\frac{M}{2}\left(\frac{\partial\mathbf{R}_a(u)}{\partial u}\right)^2 + \frac{\lambda}{2}\mathbf{R}_a^2(u)\right]$$
$$+ \frac{1}{2L}\sum_{ab} p_{ab}\int_0^L du\int_0^L du'\mathbf{R}_a(u)\cdot\mathbf{R}_b(u'),$$
(2.11)

with

$$p_{ab} = \begin{pmatrix} \lambda_1-\lambda & -s & \cdots & -s \\ -s & \lambda_1-\lambda & \ddots & \vdots \\ \vdots & \ddots & \ddots & -s \\ -s & \cdots & -s & \lambda_1-\lambda \end{pmatrix},$$
(2.12)

which reduces to the EM variational Hamiltonian if $p_{ab}=0$ (i.e., if $\lambda_1=\lambda$ and $s=0$). For now we consider a replica symmetric parametrization. We will discuss a possible replica symmetry breaking parametrization later on. Using this parametrization of h_n our task is to calculate the free energy from Eq. (2.6). This is achieved by first writing down the propagator associated with βh_n:

$$G_{ab}(\omega) = \{\beta[(M\omega^2+\mu)\mathbf{1} - \tilde{\mathbf{q}}(\omega)]\}^{-1}_{ab}$$
(2.13)

with

$$\tilde{q}_{ab}(\omega) = \int_0^L du\, q_{ab}(u)\exp(-i\omega u).$$
(2.14)

For the function $q_{ab}(u)$ given by Eq. (2.8) we find

$$\tilde{q}_{ab}(\omega) = -\delta_{ab}[(\lambda - \mu) + (\lambda_1 - \lambda)\delta_{\omega,0}] + (1 - \delta_{ab})s\,\delta_{\omega,0},$$
$$(2.15)$$

and thus

$$G_{ab}(\omega) = \beta^{-1}\{[M\omega^2 + \lambda + (\lambda_1 - \lambda + s)\delta_{\omega,0}]\mathbf{1} - s\,\delta_{\omega,0}\}_{ab}^{-1},$$
$$(2.16)$$

which gives after inverting an $n \times n$ matrix and taking the limit $n \to 0$,

$$\beta G_{ab}(\omega=0) = \frac{\lambda_1 + 2s}{(\lambda_1 + s)^2}\delta_{ab} + \frac{s}{(\lambda_1 + s)^2}(1 - \delta_{ab}),$$
$$(2.17)$$

$$\beta G_{ab}(\omega \neq 0) = \frac{1}{M\omega^2 + \lambda}\delta_{ab}.$$
$$(2.18)$$

Since the interval on which u is defined is finite ($0 \leq u \leq L$), the ''frequencies'' ω are discrete and satisfy

$$\omega_m = \frac{2\pi}{L}m, \quad m = 0, \pm 1, \pm 2, \ldots .$$
$$(2.19)$$

We can now use the fact that

$$\langle \mathbf{R}_a(u) \cdot \mathbf{R}_b(u') \rangle \equiv d g_{ab}(u - u') = \frac{d}{L}\sum_\omega e^{-i\omega(u-u')}G_{ab}(\omega),$$
$$(2.20)$$

to obtain an expression for the correlation function of interest and for the free energy. For the correlation function we obtain

$$C(\ell) = \frac{1}{d}\langle\langle [\mathbf{R}(\ell) - \mathbf{R}(0)]^2 \rangle\rangle_R$$

$$= \frac{1}{nd}\sum_{a=1}^{n}\langle [\mathbf{R}_a(\ell) - \mathbf{R}_a(0)]^2 \rangle$$

$$= \frac{2}{nL}\sum_{a=1}^{n}\sum_\omega G_{aa}(\omega)(1 - e^{-i\omega\ell})$$

$$= \frac{2}{\beta L}\sum_{\omega \neq 0}\frac{1 - e^{-i\omega\ell}}{M\omega^2 + \lambda}.$$
$$(2.21)$$

For the free energy we find from Eq. (2.6)

$$\frac{n\langle F \rangle}{d} = \text{const} + \frac{1}{2}(\mu - \lambda)\sum_{a=1}^{n}\sum_\omega G_{aa}(\omega)$$

$$- \frac{1}{2}\sum_{ab}p_{ab}G_{ab}(\omega=0) - \frac{1}{2\beta}\sum_\omega \text{tr}\ln\mathbf{G}(\omega)$$

$$- \frac{\beta g L}{2d}\int_0^L dz \int \frac{d\mathbf{k}}{(2\pi)^d}$$

$$\times \sum_{ab}\exp\left(-\frac{\mathbf{k}^2}{2L}\sum_\omega [G_{aa}(\omega) + G_{bb}(\omega)\right.$$

$$\left. -2e^{-i\omega z}G_{ab}(\omega)]\right).$$
$$(2.22)$$

We now use the formula [see, e.g., Gradshteyn and Ryzhik [14], Eq. (1.445.2)]

$$\frac{1}{L}\sum_\omega \frac{e^{-i\omega z}}{M\omega^2 + \lambda} = \frac{1}{2\sqrt{M\lambda}}\frac{\cosh[\alpha(1 - 2z/L)]}{\sinh(\alpha)},$$

$$\alpha = \frac{L}{2}\sqrt{\frac{\lambda}{M}}, \quad 0 \leq z \leq L$$
$$(2.23)$$

to calculate the correlation function and the free energy in the limit $n \to 0$: For the correlation function we obtain

$$C(\ell) = \frac{1}{\beta\sqrt{M\lambda}}$$

$$\times \left(\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}} - \frac{\cosh[L\sqrt{\lambda/M}(1 - 2\ell/L)/2]}{\sinh(L\sqrt{\lambda/M}/2)} \right)$$
$$(2.24)$$

and for the free energy

$$\frac{\beta\langle F \rangle}{Ld} = \text{const} + \frac{(\mu - \lambda)}{4\sqrt{M\lambda}}\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}} + \frac{\mu}{2L}\left(\frac{\lambda_1 + 2s}{(\lambda_1 + s)^2} - \frac{1}{\lambda} \right)$$

$$+ \frac{1}{L}\ln\sinh\frac{L}{2}\sqrt{\frac{\lambda}{M}} + \frac{1}{2L}\ln\left(1 + \frac{s}{\lambda_1} \right) + \frac{1}{2L}\ln\frac{\lambda_1}{\lambda}$$

$$- \frac{1}{2L}\frac{s}{\lambda_1 + s} - \frac{\beta^2 g}{2d}\int_0^L dz \int \frac{d\mathbf{k}}{(2\pi)^d}[\exp(-\mathbf{k}^2 a_1)$$

$$- \exp(-\mathbf{k}^2 a_2)],$$
$$(2.25)$$

with

$$a_1 = \frac{1}{2\beta\sqrt{M\lambda}}\left(\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}} - \frac{\cosh[L\sqrt{\lambda/M}(1 - 2z/L)/2]}{\sinh(L\sqrt{\lambda/M}/2)} \right),$$
$$(2.26)$$

$$a_2 = \frac{1}{\beta L}\left(\frac{1}{\lambda_1 + s} - \frac{1}{\lambda} \right) + \frac{1}{2\beta\sqrt{M\lambda}}\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}}.$$
$$(2.27)$$

Some of the details of the calculation are given in the Appendix. The constant term does not depend on the variational parameters. So far the calculation has been exact but now we are interested in the large L limit. Before we proceed it will be instructive to pause to review the calculation of EM who have chosen $\mathbf{p} = 0$, i.e., $s = 0$ and $\lambda_1 = \lambda$ (recall that in their notation $\lambda \propto q^2$). They also take $\mu = 0$. In that case the free energy simplifies to give

$$\frac{\beta\langle F \rangle}{Ld} = \text{const} - \frac{\lambda}{4\sqrt{M\lambda}}\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}} + \frac{1}{L}\ln\sinh\frac{L}{2}\sqrt{\frac{\lambda}{M}}$$

$$- \frac{\beta^2 g}{d}\int_0^{L/2} dz \int \frac{d\mathbf{k}}{(2\pi)^d}[\exp(-\mathbf{k}^2 a_1)$$

$$- \exp(-\mathbf{k}^2 a_2)],$$
$$(2.28)$$

with a_1 still given by Eq. (2.26) and

$$a_2 = \frac{1}{2\beta\sqrt{M\lambda}} \coth \frac{L}{2}\sqrt{\frac{\lambda}{M}}. \qquad (2.29)$$

We also noticed that since a_1 is symmetric about the point $z = L/2$ we have limited the z integration up to $L/2$ and multiplied the integral by 2. We can now take the limit of large L. It is at the point $z = L/2$ that the integrand vanishes for large L. In this limit we find (upon dropping the constant):

$$\frac{\beta\langle F\rangle}{Ld} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{\beta^2 g}{d}\int_0^\infty dz \int \frac{d\mathbf{k}}{(2\pi)^d} \\ \times [\exp(-\mathbf{k}^2 a_1) - \exp(-\mathbf{k}^2 a_2)], \qquad (2.30)$$

with

$$a_1 = \frac{1}{2\beta\sqrt{M\lambda}}[1 - \exp(-z\sqrt{\lambda/M})], \quad a_2 = \frac{1}{2\beta\sqrt{M\lambda}}. \qquad (2.31)$$

Notice that the factor of 2 in front of the integral due to the aforementioned symmetry was missed in Ref. [1]. This is of no importance since it just renormalizes the strength of the disorder. The integral over $\mathbf{k}$ can now be done to yield

$$\frac{\beta\langle F\rangle}{Ld} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{\beta^2 g}{d}\left(\frac{\beta\sqrt{M\lambda}}{2\pi}\right)^{d/2}\int_0^\infty dz \\ \times \left[\frac{1}{[1 - \exp(-z\sqrt{\lambda/M})]^{d/2}} - 1\right]. \qquad (2.32)$$

At this point we realize that the z integral is infrared divergent for any dimension $d \geq 2$. We can trace this back to the short distance singularity of the Dirac delta function correlation. We thus replace the delta function by a regularized form:

$$\delta^{(d)}(\mathbf{R}) \to \frac{1}{(\pi d\xi^2)^{d/2}}\exp\left(-\frac{\mathbf{R}^2}{d\xi^2}\right), \qquad (2.33)$$

where ξ is small (we can think of it as the intrinsic diameter of the polymer thread). Using the representation given by Eq. (2.4) for the right-hand side and carrying out the $\mathbf{y}$ integration yields

$$\frac{\beta\langle F\rangle}{Ld} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{\beta^2 g}{d}\int_0^\infty dz \int \frac{d\mathbf{k}}{(2\pi)^d}\exp\left(-\frac{d}{4}\xi^2\mathbf{k}^2\right) \\ \times [\exp(-\mathbf{k}^2 a_1) - \exp(-\mathbf{k}^2 a_2)], \qquad (2.34)$$

and the integrals are now properly regularized to yield a finite expression. To find the optimum variational parameter λ, we take the derivative of the above expression with respect to $\sqrt{\lambda}$:

$$1 = \frac{2\beta g}{d\lambda}\sqrt{\frac{M}{\lambda}}\int_0^\infty d\tau \int \frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\exp\left(-\frac{d}{4}\xi^2\mathbf{k}^2\right) \\ \times \{[1 - \exp(-\tau) - \tau\exp(-\tau)]\exp(-\mathbf{k}^2 a_1) \\ - \exp(-\mathbf{k}^2 a_2)\}, \qquad (2.35)$$

and the z variable has been rescaled by $z \to \tau\sqrt{M/\lambda}$. The $\mathbf{k}$ integration can now be done to yield

$$1 = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}}\lambda^{(d-4)/4}\int_0^\infty d\tau \\ \times \left\{\frac{1 - e^{-\tau} - \tau e^{-\tau}}{(1 - e^{-\tau} + \Delta)^{d/2+1}} - \frac{1}{(1+\Delta)^{d/2+1}}\right\}, \qquad (2.36)$$

with $\Delta = \xi^2 d\beta\sqrt{M\lambda}/2$. At this point we see that the integral is finite for $d < 4$ even in the limit $\Delta \to 0$ (which follows from $\xi \to 0$). Let us denote the integral in this limit by I_d:

$$I_d = \int_0^\infty d\tau\left\{\frac{1}{(1 - e^{-\tau})^{d/2}} - \frac{\tau e^{-\tau}}{(1 - e^{-\tau})^{d/2+1}} - 1\right\}, \qquad (2.37)$$

so, Eq. (2.36) becomes

$$1 = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}}I_d\lambda^{(d-4)/4}. \qquad (2.38)$$

Unfortunately EM did not realize that this integral is *negative* for $d = 3$ (and also $d = 2$). The indefinite integral can be carried out analytically (e.g., using MATHEMATICA) in $d = 3$ to give

$$I_3(\tau) = -\tau + \frac{2}{3}\sqrt{1 - e^{-\tau}}\left(-\frac{1}{1 - e^{-\tau}} + \frac{\tau}{(1 - e^{-\tau})^2}\right) \\ + \frac{1}{3}\ln\frac{1 + \sqrt{1 - e^{-\tau}}}{1 - \sqrt{1 - e^{-\tau}}},$$

$$I_3 = \lim_{\tau\to\infty}I(\tau) - \lim_{\tau\to 0}I(\tau) = -\frac{2}{3}(1 - \ln 2) \approx -0.20457. \qquad (2.39)$$

[In $d = 2$, one has $I_2(\tau) = \tau/(e^\tau - 1)$, and $I_2 = -1$.] From here it follows that Eq. (2.38) *has no solution for* λ. This is a very important observation. Notice that all fractional powers of λ are always to be taken as positive. For example, in the integral

$$\int \frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2 \exp(-\mathbf{k}^2 a_2) = \int \frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2$$

$$\times \exp\left(-\mathbf{k}^2 \frac{1}{2\beta\sqrt{M\lambda}}\right)$$

$$= \frac{d}{2^{d+1}\pi^{d/2}}(2\beta\sqrt{M\lambda})^{d/2+1}, \tag{2.40}$$

which is part of the result derived above, $\sqrt{\lambda}$ in the integrand is positive, and so must be the result of the integration since the integrand is positive definite. There is no way to argue that $\lambda^{1/4}$ can be taken as the negative square root of $\sqrt{\lambda}$. To elucidate further the fact that there is no value of λ which extremize the variational free energy we return to Eq. (2.34) of the free energy and carry out the $\mathbf{k}$ integration to find for $d=3$

$$\frac{\beta\langle F\rangle}{3L} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{g\beta^{7/2}M^{5/4}\lambda^{1/4}}{3(2\pi)^{3/2}}\int_0^\infty d\tau$$

$$\times\left(\frac{1}{[1+\Delta-\exp(-\tau)]^{3/2}} - \frac{1}{(1+\Delta)^{3/2}}\right), \tag{2.41}$$

with $\Delta = \xi^2 d\beta\sqrt{M\lambda}/2$. Again the integral can be done analytically (MATHEMATICA) and we find

$$\frac{\beta\langle F\rangle}{3L} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{g\beta^{7/2}M^{5/4}\lambda^{1/4}}{3(2\pi)^{3/2}}$$

$$\times\left(-2(1-\ln 2) + \frac{2}{\sqrt{\Delta}} + O(\sqrt{\Delta})\right), \tag{2.42}$$

substituting for Δ one obtains

$$\frac{\beta\langle F\rangle}{3L} = -\frac{g\beta^3 M}{(3\pi)^{3/2}\xi} + \frac{1}{4}\sqrt{\frac{\lambda}{M}}$$

$$+ (1-\ln 2)\frac{2g\beta^{7/2}M^{5/4}\lambda^{1/4}}{3(2\pi)^{3/2}} + O(\xi). \tag{2.43}$$

We see that the divergent term (as $\xi\to 0$) is independent of λ. We also see that the free energy is a monotonically increasing function of λ and thus has no extrema as a function of it. Derivative of the last expression with respect to $\sqrt{\lambda}$ agrees with our previous result. Thus we see that the one parameter variational Hamiltonian does not yield a meaningful result.

Let us now return to the more general expression for the variational free energy given in Eq. (2.25). Before we consider the large L limit we can draw a general conclusion. Let us calculate the derivative of the fee energy with respect to λ_1 and s:

$$\frac{-\mu\lambda_1 - 3\mu s + 3s\lambda_1 + 2s^2 + \lambda_1^2}{2L(\lambda_1+s)^3}$$

$$= \left(\frac{\partial a_2}{\partial\lambda_1}\right)\frac{\beta^2 g}{2d}\int_0^L dz\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\exp(-\mathbf{k}^2 a_2), \tag{2.44}$$

$$\frac{s(2\mu-\lambda_1-s)}{2L(\lambda_1+s)^3} = \left(\frac{\partial a_2}{\partial s}\right)\frac{\beta^2 g}{2d}\int_0^L dz\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\exp(-\mathbf{k}^2 a_2). \tag{2.45}$$

Since

$$\left(\frac{\partial a_2}{\partial\lambda_1}\right) + \left(\frac{\partial a_2}{\partial s}\right) = 0, \tag{2.46}$$

we find that upon adding the two equations we get

$$\frac{\lambda_1 - \mu + s}{2L(\lambda_1+s)^2} = 0, \tag{2.47}$$

which implies

$$\lambda_1 + s = \mu. \tag{2.48}$$

This is an important general result. Substituting this result in Eq. (2.44) we find

$$s = \frac{\beta g}{d}L\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\exp(-\mathbf{k}^2 a_2) = \frac{2\pi\beta gL}{(4\pi a_2)^{d/2+1}}, \tag{2.49}$$

with

$$a_2 = \frac{1}{\beta L}\left(\frac{1}{\mu} - \frac{1}{\lambda}\right) + \frac{1}{2\beta\sqrt{M\lambda}}\coth\frac{L}{2}\sqrt{\frac{\lambda}{M}}, \tag{2.50}$$

which gives a relation between s and λ. However as we will see in a moment, only the combination $\lambda_1 + s = \mu$, enters in the equation for λ.

Returning to Eq. (2.25) we see that upon taking the derivative with respect to $\sqrt{\lambda}$ the only dependence on s and λ_1 is through the combination $\lambda_1 + s$ in a_2. It is simpler to take the limit of large L and to write up the resulting equation for λ up to exponentially small terms in L:

$$\lambda - \mu = \frac{4(\lambda-\mu)}{L}\sqrt{\frac{M}{\lambda}}$$

$$+ \frac{2\beta g}{d}\sqrt{\frac{M}{\lambda}}\int_0^{(L/2)\sqrt{\lambda/M}}d\tau\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2$$

$$\times\{\exp(-\mathbf{k}^2 a_1)[1-\exp(-\tau)-\tau\exp(-\tau)]$$

$$- \exp(-\mathbf{k}^2 a_2)(1-4\sqrt{M/\lambda}/L)\}, \tag{2.51}$$

with

$$a_1 = \frac{1-e^{-\tau}}{2\beta\sqrt{M\lambda}}, \quad a_2 = \frac{1}{2\beta\sqrt{M\lambda}} + \frac{1}{\beta L\mu} - \frac{1}{\beta L\lambda}. \tag{2.52}$$

If μ is finite, one can proceed with expanding $\exp(-\mathbf{k}^2 a_2)$ in powers of $1/L$ as will be done later. However, if one attempts to take the limit $\mu \to 0$ we see immediately a potential problem because of the term $1/(\beta L \mu)$ in a_2. If we carry out the $\mathbf{k}$ integration we find

$$\lambda - \mu = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}}\lambda^{d/4}\int_0^{(L/2)\sqrt{\lambda/M}} d\tau$$

$$\times \left(\frac{1-e^{-\tau}-\tau e^{-\tau}}{(1-e^{\tau})^{d/2+1}} - \frac{1}{\left(1+\frac{2}{L}\sqrt{\frac{M}{\lambda}\frac{\lambda-\mu}{\mu}}\right)^{d/2+1}} \right),$$

$$(2.53)$$

where we have omitted subleading terms in $1/L$. As $\mu \to 0$ for fixed large L, the last term in the integral vanishes (as $\lambda - \mu$ remains finite for $g \neq 0$). The integral over τ no longer converges for large L, but is rather proportional to L. To leading order we get (by subtracting and adding 1 to the integrand)

$$\lambda = \frac{g\beta^{d/2+2}M^{d/4+1/2}L}{(2\pi)^{d/2}}\lambda^{(d+2)/4}, \qquad (2.54)$$

which gives

$$\lambda = \left(\frac{g\beta^{d/2+2}M^{d/4+1/2}L}{(2\pi)^{d/2}} \right)^{4/(2-d)}. \qquad (2.55)$$

We see that the borderline dimension appears to be $d=2$. Indeed from Eq. (2.24) it follows that for large L

$$C(\ell) \approx \frac{1}{\beta\sqrt{M\lambda}}[1-\exp(-\ell\sqrt{\lambda/M})], \qquad (2.56)$$

and thus the radius of gyration satisfies

$$R \sim \lambda^{-1/4} \sim (gL)^{-1/(2-d)}, \qquad (2.57)$$

which agrees perfectly with Eq. (1.6) for $d<2$. To see what happens for $d>2$ we can easily show that in the limit $\mu \to 0$ the free energy becomes of the form

$$\frac{\beta\langle F\rangle}{Ld} = \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{\beta^2 g}{d}\left(\frac{\beta\sqrt{M\lambda}}{2\pi}\right)^{d/2}\sqrt{\frac{M}{\lambda}}\int_0^{(L/2)\sqrt{\lambda/M}} d\tau$$

$$\times \frac{1}{[1+\Delta-\exp(-\tau)]^{d/2}}, \qquad (2.58)$$

where again we regularized with $\Delta = \xi^2 d\beta\sqrt{M\lambda}/2$. This gives

$$\frac{\beta\langle F\rangle}{Ld} = \text{const} + \frac{1}{4}\sqrt{\frac{\lambda}{M}} - \frac{\beta^2 g}{2d}\left(\frac{\beta\sqrt{M\lambda}}{2\pi}\right)^{d/2}L, \quad (2.59)$$

and when using $\lambda \sim d^2\beta^{-2}M^{-1}R^{-4}$ we obtain

$$\beta\langle F\rangle = \text{const}\times L + \frac{d^2}{4\beta M}\frac{L}{R^2} - \frac{d^{d/2}\beta^2 g}{2(2\pi)^{d/2}}\frac{L^2}{R^d}, \quad (2.60)$$

which coincides with Eq. (1.4) and shows that for $d>2$, $F \to -\infty$ as $R \to 0$ and there is always collapse. Thus we see that in the limit of $\mu \to 0$ we recover the annealed result from the replica calculation as expected.

If on the other hand μ is finite, we can expand $\exp(-\mathbf{k}^2 a_2)$ in powers of $1/L$ and we find to leading order in L:

$$\lambda - \mu = \frac{2\beta g}{d}\sqrt{\frac{M}{\lambda}}\int_0^\infty d\tau\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\left[\exp\left(-\mathbf{k}^2\frac{1-e^{-\tau}}{2\beta\sqrt{M\lambda}}\right)\right.$$

$$\times [1-\exp(-\tau)-\tau\exp(-\tau)] - \exp\left(-\frac{\mathbf{k}^2}{2\beta\sqrt{M\lambda}}\right) \bigg]$$

$$+ \frac{4\beta g}{d}\sqrt{\frac{M}{\lambda}}\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^2\exp\left(-\frac{\mathbf{k}^2}{2\beta\sqrt{M\lambda}}\right)$$

$$+ \frac{g}{d}\left(\frac{1}{\mu}-\frac{1}{\lambda}\right)\int\frac{d\mathbf{k}}{(2\pi)^d}\mathbf{k}^4\exp\left(-\frac{\mathbf{k}^2}{2\beta\sqrt{M\lambda}}\right).$$

$$(2.61)$$

The last two terms are also O(1) although they originated from seemingly $1/L$ terms, since we obtain a factor of L from the range of integration over τ. Evaluating the integrals we find

$$\lambda - \mu = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}}\left(I_d+2+\frac{d+2}{2}\frac{\lambda-\mu}{\mu}\right)\lambda^{d/4}. \qquad (2.62)$$

For small g we can solve this equation in powers of g. Defining a dimensionless constant

$$\tilde{g} = \frac{g(\beta^2 M)^{(d+4)/4}\mu^{(d-4)/4}}{(2\pi)^{d/2}}, \qquad (2.63)$$

we cast the Eq. (2.62) in the form

$$h = 1 + \tilde{g}2\left(I_d + \frac{2-d}{2} + \frac{2+d}{2}h\right)h^{d/4}, \qquad (2.64)$$

with $h \equiv \lambda/\mu$. To second order in $\tilde{g}$ we find

$$\lambda/\mu = 1 + 2(I_d+2)\tilde{g} + (I_d+2)[d(I_d+2)+2(d+2)]\tilde{g}^2$$

$$+ \cdots. \qquad (2.65)$$

Thus as g increases from 0, λ is an increasing function of g starting from an initial value of μ. However, a numerical solution of Eq. (2.62) (for $d=3$) reveals that the solution becomes ill behaved as λ becomes of magnitude $\sim 2\mu$. This happens for $\tilde{g} \sim 1/[2^{7/4}(I_3+2)]$. The reason for this is as will

become evident in the next section is that the replica symmetric solution becomes invalid at this point and has to be replaced by a replica symmetry breaking solution. This will become clear in the next section where we will find the correct solution for larger values of $\tilde{g}$. It is also clear from Eq. (2.63) that for fixed g as $\mu \to 0$, $\tilde{g}$ becomes large and we will be in the region when RSB is to be used. Thus the range of applicability of the replica symmetric solution is minimal for a small value of μ.

The rest of the section can be skipped on first reading of the paper and the interested reader might continue directly to the next section discussing the RSB solution. For completeness we display here the form Eq. (2.61) takes for a general correlation of the disorder defined in Eq. (1.14). We can use the representation given in Eq. (2.4) to obtain

$$\lambda - \mu = -4\beta g \sqrt{\frac{M}{\lambda}} \int_0^\infty d\tau \left[\hat{f}' \left(\frac{1 - e^{-\tau}}{\beta \sqrt{M\lambda}} \right) \right.$$
$$\times [1 - \exp(-\tau) - \tau \exp(-\tau)] - \hat{f}' \left(\frac{1}{\beta \sqrt{M\lambda}} \right) \right]$$
$$- 8\beta g \sqrt{\frac{M}{\lambda}} \hat{f}' \left(\frac{1}{\beta \sqrt{M\lambda}} \right)$$
$$+ 4g \left(\frac{1}{\mu} - \frac{1}{\lambda} \right) \hat{f}'' \left(\frac{1}{\beta \sqrt{M\lambda}} \right), \qquad (2.66)$$

where we defined

$$\hat{f}(a) \equiv \int d\mathbf{y} f(\mathbf{y}^2/d) \int \frac{d\mathbf{k}}{(2\pi)^d} \exp(-i\mathbf{k} \cdot \mathbf{y}) \exp\left(-\frac{a\mathbf{k}^2}{2} \right)$$
$$= \frac{1}{\Gamma(d/2)} \int_0^\infty dx \, x^{d/2-1} e^{-x} f\left(\frac{2xa}{d} \right), \qquad (2.67)$$

and the primes stand for derivatives of $\hat{f}$, which can be obtained from the first line of Eq. (2.67) by taking the derivative with respect to a under the integral sign.

At this point we would like to discuss the more complete variational scheme that we used in Refs. [6,9] and show that all our conclusions concerning the limit $\mu \to 0$ follows from that scheme as well. Since the notation there was different we will translate the equations to the present notation but we will not rederive them here. What we did there was to consider a variational scheme in which we allowed the variable λ to depend on ω and we extremized the free energy with respect to each variable $\lambda(\omega)$. The propagator $G(\omega)$ defined in Eq. (2.16) now becomes

$$G_{ab}(\omega) = \beta^{-1}(\{M\omega^2 + \lambda(\omega)$$
$$+ [\lambda_1 - \lambda(0) + s]\delta_{\omega,0}\}\mathbf{1} - s\delta_{\omega,0})_{ab}^{-1}. \qquad (2.68)$$

We have found that the relation $\lambda_1 + s = \mu$ still holds and s and $\lambda(\omega)$ satisfy the equations

$$s = -2\beta g L \hat{f}'(2a_2), \qquad (2.69)$$

$$\lambda(\omega) - \mu = -s - 2\beta g \int_0^L dz (1 - e^{i\omega z}) \hat{f}'(2a_1(z)), \quad \omega \neq 0,$$
$$(2.70)$$

with

$$a_1(z) = \frac{1}{\beta L} \sum_{\omega \neq 0} \frac{1 - e^{-i\omega z}}{M\omega^2 + \lambda(\omega)}, \qquad (2.71)$$

$$a_2 = \frac{1}{\beta \mu L} + \frac{1}{\beta L} \sum_{\omega \neq 0} \frac{1}{M\omega^2 + \lambda(\omega)}. \qquad (2.72)$$

For a regularized delta function correlation we have

$$\hat{f}'(a) = -\frac{1}{2(2\pi)^{d/2}} \frac{1}{(d\xi^2/2 + a)^{d/2+1}}. \qquad (2.73)$$

In the limit $\mu \to 0$, we observe that $s \to 0$, and there is no longer a cancellation of the contributions linear in L between the two terms on the right-hand side of Eq. (2.70). Instead we get

$$\lambda(\omega) = -2\beta g L \hat{f}' \left(\frac{2}{\beta L} \sum_{\omega \neq 0} \frac{1}{M\omega^2 + \lambda(\omega)} \right), \quad (2.74)$$

which yields an ω- independent solution that for the delta correlation becomes

$$\lambda = \frac{\beta g L}{(2\pi)^{d/2}} (\beta \sqrt{M\lambda})^{d/2+1}. \qquad (2.75)$$

This result exactly coincides with Eq. (2.54) derived previously.

III. REPLICA SYMMETRY BREAKING

In the previous section we have seen that the replica symmetric solution becomes invalid for fixed amount of disorder and small harmonic constant μ. In this section we show the emergence of a different solution of the variational equation which is more adequate for our problem. But in order to take advantage of such a solution we must use a more general variational scheme. Returning to Eqs. (2.8)–(2.12), we have extended the parametrization of the matrix p_{ab} in Eq. (2.12) to allow for one-step RSB by having two off-diagonal parameters s_0 ($x < x_c$) and s_1 ($x > x_c$) together with a breaking point x_c ($0 \leq x_c \leq 1$). Here x is Parisi's replica index. For details of Parisi's RSB scheme see reviews of spin glass theory [15–17]. Thus our variational scheme includes now five parameters. A one step breaking is sufficient for the case of short range correlations of the random potential [6,18].

We were able to calculate analytically the free energy with the new parameters. Here we display the final result, the details given in the Appendix:

$$\frac{\beta \langle F \rangle}{Ld} = \frac{(\mu - \lambda)}{4\sqrt{M\lambda}} \coth \frac{L}{2} \sqrt{\frac{\lambda}{M}} + \frac{\mu}{2L} \left[\frac{1}{x_c(\lambda_1 + s_1 - \Sigma)} \right.$$

$$+ \left(1 - \frac{1}{x_c}\right) \frac{1}{\lambda_1 + s_1} + \frac{s_0}{(\lambda_1 + s_1 - \Sigma)^2} - \frac{1}{\lambda}\right]$$

$$+ \frac{1}{L} \ln \sinh \frac{L}{2} \sqrt{\frac{\lambda}{M}} + \frac{1}{2L} \ln\left(1 + \frac{s_1 - \Sigma}{\lambda_1}\right) - \frac{1}{2L}$$

$$\times \left(1 - \frac{1}{x_c}\right) \ln\left(1 - \frac{\Sigma}{\lambda_1 + s_1}\right) + \frac{1}{2L} \ln \frac{\lambda_1}{\lambda}$$

$$- \frac{1}{2L} \frac{s_0}{\lambda_1 + s_1 - \Sigma} - \frac{\beta^2 g}{2d} \int_0^L dz \int \frac{d\mathbf{k}}{(2\pi)^d}$$

$$\times [\exp(-\mathbf{k}^2 a_1) - x_c \exp(-\mathbf{k}^2 a_{2l}) - (1 - x_c)$$

$$\times \exp(-\mathbf{k}^2 a_{2b})]. \tag{3.1}$$

We introduced the notation

$$\Sigma = x_c(s_1 - s_0), \tag{3.2}$$

the variable a_1 is still given by Eq. (2.26), and we defined

$$a_{2l} = \frac{1}{\beta L} \left[\frac{1}{x_c} \frac{1}{\lambda_1 + s_1 - \Sigma} + \left(1 - \frac{1}{x_c}\right) \frac{1}{\lambda_1 + s_1} - \frac{1}{\lambda} \right]$$

$$+ \frac{1}{2\beta\sqrt{M\lambda}} \coth \frac{L}{2} \sqrt{\frac{\lambda}{M}}, \tag{3.3}$$

$$a_{2b} = \frac{1}{\beta L} \left(\frac{1}{\lambda_1 + s_1} - \frac{1}{\lambda} \right) + \frac{1}{2\beta\sqrt{M\lambda}} \coth \frac{L}{2} \sqrt{\frac{\lambda}{M}}. \tag{3.4}$$

From the free energy we are able to get the following five relations (everywhere we eliminated s_1 in favor of Σ)

$$\lambda_1 + s_0 - (1 - 1/x_c)\Sigma = \mu, \tag{3.5}$$

which replaces the relation $\lambda_1 + s = \mu$ established above for the replica symmetric solution,

$$s_0 = \frac{\beta L g}{d} \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2 \exp(-\mathbf{k}^2 a_{2l}), \tag{3.6}$$

$$\Sigma = \frac{\beta g}{d} L x_c \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2 [\exp(-\mathbf{k}^2 a_{2b}) - \exp(-\mathbf{k}^2 a_{2l})], \tag{3.7}$$

$$\frac{\Sigma}{\mu + \Sigma} - \ln\left(1 + \frac{\Sigma}{\mu}\right) = \frac{\beta g}{d} L x_c \frac{\Sigma}{\mu(\mu + \Sigma)} \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2$$

$$\times \exp(-\mathbf{k}^2 a_{2l}) + \frac{\beta^2 g}{d} (L x_c)^2 \int \frac{d\mathbf{k}}{(2\pi)^d}$$

$$\times [\exp(-\mathbf{k}^2 a_{2l}) - \exp(-\mathbf{k}^2 a_{2b})], \tag{3.8}$$

$$\lambda - \mu = \frac{2\beta g}{d} \sqrt{\frac{M}{\lambda}} \int_0^{(L/2)\sqrt{\lambda/M}} d\tau \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2$$

$$\times \{\exp(-\mathbf{k}^2 a_1)[1 - \exp(-\tau) - \tau \exp(-\tau)]$$

$$\times [-x_c \exp(-\mathbf{k}^2 a_{2l}) - (1 - x_c)\exp(-\mathbf{k}^2 a_{2b})]$$

$$\times (1 - 4\sqrt{M/\lambda}/L)\}, \tag{3.9}$$

where we defined

$$a_1 = \frac{1 - e^{-\tau}}{2\beta\sqrt{M\lambda}}, \tag{3.10}$$

$$a_{2b} = \frac{1}{2\beta\sqrt{M\lambda}} - \frac{1}{\beta L\lambda} + \frac{1}{\beta L(\mu + \Sigma)}, \tag{3.11}$$

$$a_{2l} = \frac{1}{2\beta\sqrt{M\lambda}} - \frac{1}{\beta L\lambda} + \frac{1}{\beta\mu} \frac{1}{Lx_c} + \frac{1}{\beta(\mu + \Sigma)} \left(\frac{1}{L} - \frac{1}{Lx_c} \right). \tag{3.12}$$

We have simplified some expressions assuming large L and dropped a term of order $1/L$ in Eq. (3.9).

If we denote by $y_c = Lx_c$ we realize that equations (3.7) and (3.8) can be solved for Σ and y_c of $O(1)$ with respect to L. These equations are similar for those of a classical particle in a random potential [18], except for the variable λ which does not appear there. (One can recover the equations for the classical particle by taking the limit $M \to \infty$ with L fixed. One needs to replace βL with β for a particle. This limit is not meaningful for a polymer.) For small μ we can have an approximate analytical solution:

$$\Sigma = \frac{\sqrt{gd}}{(2\pi)^{d/4}} (\beta\sqrt{M\lambda})^{d/4+1} \sqrt{|\ln \mu|}, \tag{3.13}$$

$$y_c \equiv Lx_c = \frac{1}{\beta} \sqrt{\frac{d}{g}} (2\pi)^{d/4} (\beta\sqrt{M\lambda})^{-d/4} \sqrt{|\ln \mu|}, \tag{3.14}$$

$$s_0 = \text{const}$$

$$\times g^{(2-d)/4} \beta L (\beta\sqrt{M\lambda})^{-d(d+2)/8} \mu^{d/2+1} |\ln \mu|^{(d+2)/4}. \tag{3.15}$$

An analysis of the equations (expanding in power series in Σ) shows that this solution is valid as long as the condition

$$2\beta\sqrt{M\lambda} \left(\frac{g(2+d)}{2^{2+d}\pi^{d/2}} \right)^{2/(4+d)} \mu^{-4/(4+d)} \geq 1 \tag{3.16}$$

is satisfied. This inequality can also be expressed in the form

$$\tilde{g}(d+2) \left(\frac{\lambda}{\mu} \right)^{(d+4)/4} \geq 1, \tag{3.17}$$

where $\tilde{g}$ has been defined in Eq. (2.63). When the equality holds we have $\Sigma = 0$ and $x_c = (4+d)\sqrt{M\lambda}/(2\mu L)$. This can also be verified by using this condition at the equality point in the above solutions for Σ and x_c and we see that indeed

$\Sigma \sim O(\mu)$, and $x_c \sim \sqrt{M\lambda}/(\mu L)$. Solving the equality condition given by Eq. (3.17) together with Eq. (2.64) gives

$$h = 1 + \sqrt{\frac{4 + 2I_d}{d+2}} \approx 1.85 \quad \text{for} \quad d = 3, \qquad (3.18)$$

$$\tilde{g} \approx [(d+2)h^{(d+4)/4}]^{-1} \approx 0.068 \quad \text{for} \quad d = 3 \quad (3.19)$$

in agreement with our numerical solution of Eq. (3.17) which broke down for $h \approx 2$ for $d = 3$. So the point when the replica symmetric solution has to be replaced by the RSB solution is just below the point that the RS solution becomes ill behaved.

If on the other hand μ is small but fixed we can use the solution we have obtained above in the equation for λ, in the limit of large L. We obtain

$$\lambda - \mu = \frac{2\beta g}{d} \sqrt{\frac{M}{\lambda}} \int_0^\infty d\tau \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2 \{\exp[-\mathbf{k}^2 a_0 (1 - e^{-\tau})]$$

$$\times [1 - \exp(-\tau) - \tau \exp(-\tau)] - \exp(-\mathbf{k}^2 a_0)\}$$

$$- \frac{\beta g y_c}{d} \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2$$

$$\times \left\{ \exp\left[-\mathbf{k}^2 \left(a_0 + \frac{\Sigma}{\beta y_c \mu(\mu + \Sigma)} \right) \right] - \exp(-\mathbf{k}^2 a_0) \right\}$$

$$+ \frac{4\beta g}{d} \sqrt{\frac{M}{\lambda}} \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^2 \exp(-\mathbf{k}^2 a_0)$$

$$+ \frac{g}{d} \left(\frac{1}{\mu + \Sigma} - \frac{1}{\lambda} \right) \int \frac{d\mathbf{k}}{(2\pi)^d} \mathbf{k}^4 \exp(-\mathbf{k}^2 a_0), \quad (3.20)$$

where we defined

$$a_0 \equiv 1/(2\beta\sqrt{M\lambda}). \qquad (3.21)$$

We can check that for $\Sigma = 0$ it reduces to the replica symmetric equation. Carrying out the integrals we get

$$\lambda - \mu = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}} \left\{ I_d + 2 + \frac{y_c}{2} \sqrt{\frac{\lambda}{M}} \right.$$

$$\times \left[1 - \left(1 + \frac{2\Sigma\sqrt{M\lambda}}{y_c \mu(\mu + \Sigma)} \right)^{-d/2-1} \right]$$

$$\left. + \frac{d+2}{2} \frac{\lambda - \mu - \Sigma}{\mu + \Sigma} \right\} \lambda^{d/4}, \qquad (3.22)$$

and we have to substitute for Σ and y_c (which are functions of λ) from Eqs. (3.13) and (3.14), respectively. If we now consider the case of strong disorder we can neglect μ relative to λ and Σ and the above equation simplifies to give

$$\lambda^{1-d/4} = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}}$$

$$\times \left[I_d + 2 + \frac{y_c}{2} \sqrt{\frac{\lambda}{M}} + \frac{d+2}{2} \left(\frac{\lambda}{\Sigma} - 1 \right) \right]. \qquad (3.23)$$

Substituting for Σ and y_c we find

$$\lambda^{(4-d)/4} = \frac{2g\beta^{d/2+2}M^{d/4+1}}{(2\pi)^{d/2}} \left[I_d + \frac{2-d}{2} \right.$$

$$+ \frac{\sqrt{d/4}}{\sqrt{g}} (2\pi)^{1/4d} \beta^{-(4+d)/4} M^{-(4+d)/8} \lambda^{(4-d)/8}$$

$$\left. \times \sqrt{|\ln \mu|} \left(1 + \frac{d+2}{d|\ln \mu|} \right) \right]. \qquad (3.24)$$

Let us seek a solution of the form

$$\lambda = C^{8/(4-d)} (\beta^2 M)^{(4+d)/(4-d)} g^{4/(4-d)}. \qquad (3.25)$$

Substituting in Eq. (3.24) we obtain a quadratic equation for C and to leading order as $\mu \to 0$ we find

$$\lambda = \frac{d^{4/(4-d)}}{(2\pi)^{2d/(4-d)}} (\beta^2 M)^{(4+d)/(4-d)} (g|\ln \mu|)^{4/(4-d)}. \qquad (3.26)$$

Using this result inside the parentheses in Eq. (3.24) we see that we get $I_d + 2 + \frac{1}{2}d|\ln \mu|$. This shows that we were justified *a posteriori* in neglecting the constant terms. It also shows that the negative constant I_d of EM [see Eq. (2.38)] has been replaced by the term $\frac{1}{2}d|\ln \mu|$. From this final result we obtain the radius of gyration

$$R \sim (\beta^2 M\lambda/d^2)^{-1/4} = \left(\frac{d^{(d-2)/2}}{(2\pi)^{d/2}} \beta^4 M^2 g \ |\ln \mu| \right)^{-1/(4-d)}$$

$$\sim \left(\frac{4d^{d/2}}{(2\pi)^{d/2} b^4} \beta^2 g \ln V \right)^{-1/(4-d)}. \qquad (3.27)$$

This is the main result of the paper. It recovers the EM result but with their constant I_d being replaced by $2 \ln V$ as has been argued by Cates and Ball [2]. Note that we have replaced M in favor of the bond step b.

Substituting the result (3.26) obtained for λ in Eqs. (3.13) and (3.14) we find

$$\Sigma = \left(\frac{d}{(2\pi)^{d/2}} g\beta^{(d+4)/2} M^{(d+4)/4} |\ln \mu| \right)^{4/(4-d)}, \qquad (3.28)$$

$$y_c = Lx_c = \left(\frac{d^{d-2}}{(2\pi)^d} g^2 \beta^{d+4} M^d |\ln \mu|^{d-2} \right)^{-1/(4-d)}. \qquad (3.29)$$

The second equation is important since x_c can not exceed 1 (Parisi's variable x must satisfy $0 \leq x \leq 1$ [15]). For $2 < d$

<4 we see that x_c actually decreases when μ becomes small so there is no problem. Also for $d=2$ there is no problem since for large enough L, x_c is also within range. On the other hand when $d<2$, x_c increases when μ becomes small (or equivalently V becomes large) and eventually will exceed 1. For example for $d=1$ we see that this happens for $|\ln \mu| \sim g^2 L^3$, which corresponds to an extremely large volume $V' \sim \exp(g^2 L^3)$ when L is large. For $V>V'$ we revert to the annealed result, which for $d<2$ predict $R \sim (Lg)^{1/(d-2)}$ as was shown in the last section. In the large L limit this again leads to a fully collapsed polymer.

We have also verified that to leading order the free energy is given by Eq. (1.10) (there is a subleading term of the form Lg/R^{d-2} that can be neglected). It is interesting that the condition $x_c<1$ that we have applied above has a physical significance [2]. The attractive term in the free energy is [see Eq. (1.10)] of the form $-L\sqrt{g \ln V/R^d}$. This represents (up to a sign) the binding energy of the chain. In order that the polymer will be confined to a small single region of size R as given above in Eq. (3.27), the binding energy should not exceed the translational entropy $\sim \ln V$. The condition

$$\ln V < L\sqrt{g \ln V/R^d} \qquad (3.30)$$

is equivalent (up to some irrelevant constants) to the condition $x_c<1$ as can be verified by using the result (3.27) in Eq. (3.30).

IV. CONCLUSIONS

We have considered the problem of a polymer (a Gaussian chain) in a quenched disordered medium. The problem maps also to a quantum particle in a random potential, and in the presence of an additional confining harmonic force (of spring constant μ) it maps also to the problem of a flux line in a cage potential and random columnar disorder. We carried out a replica calculation in the presence of a confining harmonic force, and succeeded to ''improve'' the previous results of EM [1], in the sense that the (unphysical) constant is replaced by $\ln V$ in the equation for the variational parameter λ and hence also in the dependence of the radius of gyration on the strength of the disorder. Of course our calculation does not diminish the accomplishments of EM who pioneered the use of the variational method in the context of the replica calculation and for the first time obtained the correct scaling exponent for the dependence of the radius of gyration on the disorder for finite systems. In the infinite volume limit the chain collapses since it can find a very deep potential minimum somewhere which can accommodate it. For $2 \leq d \leq 4$ the chain is *localized* in the sense [2] that even in the large V limit two long chains introduced into the system will find the same small neighborhood to occupy (with a probability approaching 1 for large L). This is a consequence of the off diagonal spin-glass order parameter we introduced that measures overlap between different replicas. It is comforting to find out that the replica calculation can reproduce all the physical arguments introduced so cleverly by CB [2].

APPENDIX

Here we give some of the intermediate steps leading to Eqs. (2.6) and (2.22). To evaluate the expectation value of the last term in H_n it is most useful to write

$$\langle \exp i\mathbf{k}[\mathbf{R}_a(u) - \mathbf{R}_b(u')]\rangle_{h_n}$$

$$= \int \mathcal{D}\mathbf{R}_1 \cdots \mathcal{D}\mathbf{R}_n \exp\left(\sum_c \int_0^L dv\mathbf{V}_c(v) \cdot \mathbf{R}_c(v)\right.$$

$$\left. -\frac{1}{2}\sum_{cd} \int_0^L dv \int_0^L dv' \mathbf{R}_c(v) g_{cd}^{-1}(v-v')\mathbf{R}_d(v')\right)$$

$$\times \left[\int \mathcal{D}\mathbf{R}_1 \cdots \mathcal{D}\mathbf{R}_n \exp\left(-\frac{1}{2}\sum_{cd} \int_0^L dv \int_0^L dv'\right.\right.$$

$$\left.\left. \times \mathbf{R}_c(v) g_{cd}^{-1}(v-v')\mathbf{R}_d(v')\right)\right]^{-1}$$

$$= \exp\left(\frac{1}{2}\sum_{cd} \int dv \int dv' \mathbf{V}_c(v) g_{cd}(v-v')\mathbf{V}_d(v')\right)$$

$$= \exp\left(-\frac{1}{2}\mathbf{k}^2[g_{aa}(0) + g_{bb}(0) - 2g_{ab}(u-u')]\right),$$

$$(A1)$$

where

$$\mathbf{V}_c(v) = i\mathbf{k}[\delta_{c,a}\delta(v-u) - \delta_{c,b}\delta(v-u')]. \qquad (A2)$$

Next we show how to evaluate other contribution to the free energy:

$$\sum_\omega \beta G_{aa}(\omega) = \sum_{\omega \neq 0} \frac{1}{M\omega^2+\lambda} + \frac{\lambda_1+2s}{(\lambda_1+s)^2}$$

$$= \frac{L}{2\sqrt{M\lambda}} \coth\frac{L}{2}\sqrt{\frac{\lambda}{M}} - \frac{1}{\lambda} + \frac{\lambda_1+2s}{(\lambda_1+s)^2}. \qquad (A3)$$

Also

$$-\frac{1}{2n}\sum_{ab} p_{ab}\beta G_{ab}(\omega=0) = -\frac{1}{2n}\text{Tr}\,\mathbf{p}\,\mathbf{G}(0)$$

$$= \frac{\lambda(\lambda_1+2s)}{2(\lambda_1+s)^2} - \frac{1}{2}, \qquad (A4)$$

and

$$-\frac{1}{2}\sum_\omega \text{tr}\ln\beta\mathbf{G}(\omega)=\frac{1}{2}\sum_\omega \text{tr}\ln[\beta^{-1}\mathbf{G}^{-1}(\omega)]$$

$$=\frac{n}{2}\sum_\omega \ln(M\omega^2+\lambda)-\frac{n}{2}\ln\lambda$$

$$+\frac{1}{2}\text{tr}\ln[\beta^{-1}\mathbf{G}^{-1}(0)], \qquad (A5)$$

but

$$\frac{n}{2}\sum_\omega \ln(M\omega^2+\lambda)=n\ln\left(2\sinh\frac{L}{2}\sqrt{\frac{\lambda}{M}}\right)+n\,\text{const} \qquad (A6)$$

[see, e.g., Ref. [14], p. 44, Eq. (1.431.2)]. The constant term (which is infinite) is eliminated by the normalization of the functional integral, and in any case does not depend on λ. Also

$$\text{tr}\ln(\beta^{-1}\mathbf{G}^{-1}(0))=\text{tr}\ln\begin{pmatrix}\lambda_1 & -s & \cdots & -s\\ -s & \lambda_1 & \ddots & \vdots\\ \vdots & \ddots & \ddots & -s\\ -s & \cdots & -s & \lambda_1\end{pmatrix}=n\ln\lambda_1$$

$$+n\ln\left(1+\frac{s}{\lambda_1}\right)-n\frac{s}{\lambda_1+s}+o(n^2). \qquad (A7)$$

For the case of one-step RSB we have to calculate the propagator by inverting Parisi type matrices. It is helpful to use formulas found in an appendix of Ref. [12]. We find

$$\beta G_{aa}(\omega=0)=\frac{1}{x_c(\lambda_1+s_1-\Sigma)}+\left(1-\frac{1}{x_c}\right)\frac{1}{\lambda_1+s_1}$$

$$+\frac{s_0}{(\lambda_1+s_1-\Sigma)^2}, \qquad (A8)$$

$$\beta G(\omega=0,x)=\frac{s_0}{(\lambda_1+s_1-\Sigma)^2}, \quad x<x_c, \qquad (A9)$$

$$\beta G(\omega=0,x)=\frac{1}{x_c(\lambda_1+s_1-\Sigma)}-\frac{1}{x_c}\frac{1}{\lambda_1+s_1}$$

$$+\frac{s_0}{(\lambda_1+s_1-\Sigma)^2}, \quad x>x_c, \qquad (A10)$$

$$\beta G_{ab}(\omega\neq0)=\frac{1}{M\omega^2+\lambda}\delta_{ab}, \qquad (A11)$$

where x is Parisi's index on the interval $[0,1]$.

Next we show how various other contribution to the free energy become in the RSB case:

$$\frac{1}{n}\sum_a\sum_\omega \beta G_{aa}(\omega)$$

$$=\sum_{\omega\neq0}\frac{1}{M\omega^2+\lambda}+\frac{1}{n}\sum_a\sum_\omega \beta G_{aa}(\omega=0), \qquad (A12)$$

$$-\frac{1}{2n}\sum_{ab}p_{ab}\beta G_{ab}(\omega=0)$$

$$=-\frac{1}{2n}\text{Tr}\,\mathbf{p}\,\mathbf{G}(0)=\frac{\lambda}{2}\beta G_{aa}(\omega=0)-\frac{1}{2}, \qquad (A13)$$

and finally

$$-\frac{1}{2n}\sum_\omega \text{tr}\ln\beta\mathbf{G}(\omega)$$

$$=\frac{1}{2n}\sum_\omega \text{tr}\ln[\beta^{-1}\mathbf{G}^{-1}(\omega)]=\frac{1}{2}\sum_\omega \ln(M\omega^2+\lambda)$$

$$-\frac{1}{2}\ln\lambda+\frac{1}{2n}\text{tr}\ln[\beta^{-1}\mathbf{G}^{-1}(0)]=\ln\left(2\sinh\frac{L}{2}\sqrt{\frac{\lambda}{M}}\right)$$

$$+\frac{1}{2}\ln\frac{\lambda_1}{\lambda}+\frac{1}{2}\ln\left(1+\frac{s_1-\Sigma}{\lambda_1}\right)-\frac{1}{2}\left(1-\frac{1}{x_c}\right)$$

$$\times\ln\left(1-\frac{\Sigma}{\lambda_1+s_1}\right)-\frac{1}{2}\frac{s_0}{\lambda_1+s_1-\Sigma}+\text{const}+o(n). \qquad (A14)$$

The coefficient a_1 in the exponential is given as before by

$$a_1=\frac{1}{L}\sum_{\omega\neq0}G_{aa}(\omega)(1-e^{i\omega z}), \qquad (A15)$$

and a_2 becomes

$$a_2(x)=\frac{1}{L}[G_{aa}(\omega=0)-G(\omega=0,x)]+\frac{1}{L}\sum_{\omega\neq0}G_{aa}(\omega)$$

$$=a_{2l}, \quad x<x_c \qquad (A16)$$

$$=a_{2b}, \quad x>x, \qquad (A17)$$

and the explicit expressions for a_1, a_{2l} and a_{2b} are given in Eqs. (2.26), (3.3), and (3.4), respectively. Notice also that

$$\frac{1}{n}\sum_{a\neq b}\exp[-\mathbf{k}^2 a_2(x)]=-\int_0^1 dx\exp[-\mathbf{k}^2 a_2(x)]$$

$$=-x_c\exp(-\mathbf{k}^2 a_{2l})-(1-x_c)$$

$$\times\exp(-\mathbf{k}^2 a_{2b}). \qquad (A18)$$

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